1 RATIO AND PROPORTION, INDICES, LOGARITHMS

THIS CHAPTER INCLUDES

- Ratio
- Proportion
- Indices
- Logarithm

CHAPTER AT A GLANCE

Topic	Important Highlight
Ratio	A ratio is a comparison of the sizes of two or more quantities of the same kind by division. If a and b are two quantities of the same kind (in same units), then the fraction a/b is called the ratio of a to b. It is written as a : b. Thus, the ratio of a to b = a/b or a : b. The quantities a and b are called the terms of the ratio, a is called the first term or antecedent and b is called the second term or consequent. • Both terms of a ratio can be multiplied or divided by the same (non - zero) number. • Usually, a ratio is expressed in lowest terms (or simplest form). • The order of the terms in a ratio is important. • Ratio exists only between quantities of the same kind. • Quantities to be compared (by division) must be in the same units.

	• To compare two ratios, convert them into equivalent like fractions. • If a quantity increases or decreases in the ratio a : b then new quantity = b of the original quantity/a The fraction by which the original quantity is multiplied to get a new quantity is called the factor multiplying ratio.
Inverse Ratio	One ratio is the inverse of another if their product is 1.Thus a : b is the inverse of b : a and *vice versa*. 1. A ratio a : b is said to be of greater inequality if a>b and of less inequality if a<b. 2. The ratio compounded of the two ratios a : b and c : d is ac : bd. 3. A ratio compounded of itself is called its duplicate ratio. 4. The sub-duplicate ratio of a : b is $\sqrt{a}$ $\sqrt{b}$ and the sub-triplicate ratio of a : b is $\sqrt[3]{a}$ $\sqrt[3]{b}$. 5. If the ratio of two similar quantities can be expressed as a ratio of two integers, the quantities are said to be commensurable; otherwise, they are said to be incommensurable. 6. Continued Ratio is the relation (or compassion) between the magnitudes of three or more quantities of the same kind. The continued ratio of three similar quantities a, b, c is written as a : b : c.
Proportions	An equality of two ratios is called a proportion. Four quantities a, b, c, d are said to be in proportion if a : b = c : d (also written as a : b :: c : d) i.e. if a/b = c/d i.e. if ad = bc. First and fourth terms are called extremes (or extreme terms). Second and third terms are called means (or middle terms).

	If a : b = c : d then d is called fourth proportional. If a : b = c : d are in proportion then a/b = c/d i.e. ad = bc i.e. product of extremes = product of means. This is called *cross product rule*. Three quantities a, b, c of the same kind (in same units) are said to be in continuous proportion if a : b = b : c i.e. a/b = b/c i.e. b2 = ac If a, b, c are in continuous proportion, then the middle term b is called the mean proportional between a and c, a is the first proportional and c is the third proportional. Thus, if b is mean proportional between a and c, then b2 = ac i.e. b = $\sqrt{ac}$. When three or more numbers are so related that the ratio of the first to the second, the ratio of the second to the third, third to the fourth etc. are all equal, the numbers are said to be in continued proportion.
Properties of Proportion	1. If a : b = c : d, then ad = bc 2. If a : b = c : d, then b : a = d : c (Invertendo) 3. If a : b = c : d, then a : c = b : d (Alternendo) 4. If a : b = c : d, then a + b : b = c + d : d (Componendo) 5. If a : b = c : d, then a – b : b = c – d : d (Dividendo) 6. If a : b = c : d, then a + b : a – b = c + d : c – d (Componendo and Dividendo) 7. If a : b = c : d = e : f = ……..…., then each of these ratios (Addendo) is equal (a + c + e + ……..) : (b + d + f + …….)

Indices	If n is a positive integer, and 'a' is a real number, i.e. $n \in N$ and $a \in R$ (where N is the set of positive integers and R is the set of real numbers), 'a' is used to denote the continued product of n factors each equal to 'a' as shown below: $a^n = a \times a \times a$ …….. to n factors. Here a^n is a power of "a" whose base is "a" and the index or power is "n". **Law 1** $a^m \times a^n = a^{m+n}$, **Law 2** $a^m/a^n = a^{m-n}$, **Law 3** $(a^m)^n = a^{mn}$, **Law 4** $(ab)^n = a^n b^n$
Logarithms	The logarithm of a number to a given base is the index or the power to which the base must be raised to produce the number, i.e. to make it equal to the given number. If there are three quantities indicated by say a, x and n, they are related as follows: If $a^x = n$, where $n > 0$, $a > 0$ and $a \neq 1$ then x is said to be the logarithm of the number n to the base 'a' symbolically it can be expressed as follows: $\log_a n = x$ i.e. the logarithm of n to the base 'a' is x. 1. The two equations $a^x = n$ and $x = \log_a n$ are only transformations of each other and should be remembered to change one form of the relation into the other. 2. The logarithm of 1 to any base is zero. 3. The logarithm of any quantity to the same base is unity.

Fundamental Laws of Logarithm	1. Logarithm of the product of two numbers is equal to the sum of the logarithms of the numbers to the same base, i.e. $\log_a mn = \log_a m + \log_a n$ 2. The logarithm of the quotient of two numbers is equal to the difference of their logarithms to the same base, i.e. $\log_a \frac{m}{n} = \log_a m - \log_a n$ 3. Logarithm of the number raised to the power is equal to the index of the power multiplied by the logarithm of the number to the same base i.e. $\log_a m^n = n \log_a m$
Change of Base	If the logarithm of a number to any base is given, then the logarithm of the same number to any other base can be determined from the following relation. $\log_a m = \log_b m \log_a b \Rightarrow \log_b m = \frac{\log_a m}{\log_a b}$
Logarithm Tables	The logarithm of a number consists of two parts, the whole part or the integral part is called the characteristic and the decimal part is called the mantissa where the former can be known by mere inspection, the latter has to be obtained from the logarithm tables.
Characteristic	The characteristic of the logarithm of any number greater than 1 is positive and is one less than the number of digits to the left of the decimal point in the given number. The characteristic of the logarithm of any number less than one (1) is negative and numerically one more than the number of zeros to the right of the decimal point. If there is no zero then obviously it will be –1.

Mantissa	The mantissa is the fractional part of the logarithm of a given number
Antilogarithms	If x is the logarithm of a given number n with a given base then n is called the antilogarithm (antilog) of x to that base. This can be expressed as follows: If $\log_a n = x$ then n = antilog x
Relation between Indices and Logarithm	$\log_a m + \log_a n = \log_a mn$ $\log_a mn = \log_a m + \log_a n$ $\log_a m^n = n \log_a m$ $\log_b n \times \log_a b = 1$ $\log_b c \times \log_c b = l$

OBJECTIVE QUESTIONS

2006 - Nov [1] Two numbers are in the ratio 2 : 3 and the difference of their squares is 320. The numbers are :

(a) 12, 18 (b) 16, 24

(c) 14, 21 (d) None. **(1 mark)**

Answer:

(b) Let numbers be $2x$ and $3x$.

Therefore, $(3x)^2 - (2x)^2 = 320$

$9x^2 - 4x^2 = 320$

$5x^2 = 320$

$x^2 = 64$

$x = 8$

Numbers are: $2x = 2 \times 8 = 16$

$3x = 3 \times 8 = 24$

2006 - Nov [2] If p : q is the sub-duplicate ratio of $p - x^2 : q - x^2$, then x^2 is :

(a) $\frac{p}{p+q}$ (b) $\frac{q}{p+q}$

(c) $\frac{qp}{p-q}$ (d) None. **(1 mark)**

Answer:

(d) As per the given information :

$$\frac{p-x^2}{q-x^2} = \frac{P^2}{q^2}$$

$q^2 (p - x^2) = P^2(q - x^2)$

$pq^2 - x^2 q^2 = p^2 q - p^2 x^2$

$x^2 (p^2 - q^2) = pq(p - q)$

$$x^2 = \frac{pq\,(p-q)}{p^2 - q^2}$$

$$x^2 = \frac{p\,q}{p+q}$$

2006 - Nov [3] An alloy is to contain copper and zinc in the ratio 9 : 4. The zinc required to melt with 24 kg of copper is :

(a) $10\frac{2}{3}$ kg (b) $10\frac{1}{3}$ kg

(c) $9\frac{2}{3}$ kg (d) 9kg **(1 mark)**

Answer:

(a) Let the quantity of copper and zinc in an alloy be 9x kg and 4x kg.

Therefore, $9x = 24$

$$x = \frac{24}{9} = \frac{8}{3} = 2\frac{2}{3}\ kg \quad .$$

So, zinc $= 4x = 4 \times \frac{8}{3}\ kg$.

$$= 10\frac{2}{3}\ kg \quad .$$

2006 - Nov [4] $7 \log \left(\frac{16}{15}\right) + 5 \log \left(\frac{25}{24}\right) + 3 \log \left(\frac{81}{80}\right)$ is equal to :

(a) 0 (b) 1

(c) log 2 (d) log 3 **(1 mark)**

Answer:

(c) $7 \text{ Log} \left(\frac{16}{15}\right) + 5 \text{ Log} \left(\frac{25}{24}\right) + 3 \text{ Log} \left(\frac{81}{80}\right)$

$= 7(\log 16 - \log 15) + 5(\log 25 - \log 24) + 3 \log (\log 81 - \log 80)$

$= 7 [4 \log 2 - (\log 3 + \log 5)] + 5 [2 \log 5 - (3 \log 2 + \log 3)]$
$+ 3 [4 \log 3 - (4 \log 2 + \log 5)]$

$= 28 \log 2 - 7 \log 3 - 7 \log 5 + 10 \log 5 - 15 \log 2 - 5 \log 3$
$+ 12 \log 3 - 12 \log 2 - 3 \log 5 = \log 2$

2007 - Feb [5] Two numbers are in the ratio 7 : 8. If 3 is added to each of them, their ratio becomes 8 : 9. The numbers are :

(a) 14,16 (b) 24,27

(c) 21,24 (d) 16,18 **(1 mark)**

Answer:

(c) Let the numbers be $7x$ and $8x$.

So, $\frac{7x + 3}{8x + 3} = \frac{8}{9}$

$9 (7x + 3) = 8 (8x + 3)$

$63x + 27 = 64x + 24$

$x = 3$

Numbers are : $7x = 7 \times 3 = 21$

$8x = 8 \times 3 = 24$

2007 - Feb [6] A box contains ₹ 56 in the form of coins of one rupee, 50 paise and 25 paise. The number of 50 paise coin is double the number of 25 paise coins and four times the numbers of one rupee coins. The numbers of 50 paise coins in the box is :

(a) 64 (b) 32

(c) 16 (d) 14 **(1 mark)**

Answer:

(a) Let the number of one - rupee coins be x.
Then, number of 50 paise coins is $4x$
and number of 25 paise coins is $2x$
So,

$$x + \frac{4x}{2} + \frac{2x}{4} = 56$$

$$4x + 8x + 2x = 56 \times 4$$

$$14x = 224$$

$$x = \frac{224}{14} = 16$$

Number of 50 paise coins is $4 \times 16 = 64$

2007 - Feb [7] Value of $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$ is :

(a) $a + \frac{1}{a}$ (b) $a - \frac{1}{a}$

(c) $a^2 + \frac{1}{a^2}$ (d) $a^2 - \frac{1}{a^2}$ **(1 mark)**

Answer:

(b) $(a^{1/8} + a^{-1/8})(a^{1/8} - a^{-1/8})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$

$= (a^{1/4} - a^{-1/4})(a^{1/4} + a^{-1/4})(a^{1/2} + a^{-1/2})$
[using $(a^2 - b^2) = (a-b)(a+b)$]

$= (a^{1/2} - a^{-1/2})(a^{1/2} + a^{-1/2})$

$= a^1 - a^{-1}$

$= a - \frac{1}{a}$

2007 - Feb [8] The value of the expression :

$$a^{\log_a b \cdot \log_b^c \cdot \log_c^d \cdot \log_d t}$$

(a) t (b) abcdt

(c) (a + b + c + d + t) (d) None. **(1 mark)**

Answer:

(a) $a^{\log_a^b \cdot \log_b^c \cdot \log_c^d \cdot \log_d^t}$

$$a\ \frac{\log^b}{\log^a} \times \frac{\log^c}{\log^b}\ \ \frac{\log^d}{\log^c} \cdot \frac{\log^t}{\log^d}. \quad \left[\text{using } \log a^b = \frac{\log^b}{\log^a}\right] =$$

$$= a\frac{\log^t}{\log^a}$$

$$= a \log^t_a$$

$$= t \text{ [using } a^{\log_a m} = m]$$

2007 - Feb [9] If $\log_{10000} x = \frac{-1}{4}$, then x is given by:

(a) $\frac{1}{100}$ (b) $\frac{1}{10}$

(c) $\frac{1}{20}$ (d) None of these. **(1 mark)**

Answer:

(b) $\text{Log}_{1000}\ x = -\frac{1}{4}$

$(10,000)^{-1/4}$ x = [using $\log a^b = x$, $= a^x = b$

$\frac{1}{(10,000)^{1/4}} = x$

$= \frac{1}{10} = x$

2007 - May [10] Eight people are planning to share equally the cost of a rental car. If one person withdraws from the arrangement and the others share equally entire cost of the car, then the share of each of the remaining persons increased by :

(a) 1/9 (b) 1/8

(c) 1/7 (d) 7/8 **(1 mark)**

Answer:

(c) When number of people = 8

then, the share of each person = $\frac{1}{8}$ of the total cost.

When number of people = 7

then, the share of each person = $\frac{1}{7}$ of the total cost

Increase in the share of each person = $\frac{1}{7} - \frac{1}{8} = \frac{1}{56}$ i.e. $\frac{1}{7}$ *of* $\frac{1}{8}$ $\frac{1}{7}$, i.e. of the original share of each person.

2007 - May [11] A bag contains ₹ 187 in the form of 1 rupee, 50 paise and 10 paise coins in the ratio 3:4:5. Find the number of each type of coins :

(a) 102, 136, 170 (b) 136, 102, 170

(c) 170, 102, 136 (d) None. **(1 mark)**

Answer:

(a) Let the number of coins be $3x$,$4x$,and $5x$.

Then, $3x + \frac{4x}{2} + \frac{5x}{10} = 187$

$30x + 20x + 5x = 187 \times 10$

$55x = 1870$

$x = \frac{1,870}{55} = 34$

Number of coins:

One rupee = $3x = 3 \times 34 = 102$

50 paise = $4x = 4 \times 34 = 136$

10 paise = $5x = 5 \times 34 = 170$

2007 - May [12] Simplification of $\frac{x^{m+3n} \cdot x^{4m-9n}}{x^{6m-6n}}$ is :

(a) x^m (b) x^{-m}

(c) x^n (d) x^{-n} **(1 mark)**

Answer:

(b) $\frac{x^{m+3n} \cdot x^{4m-9n}}{x^{6m-6n}}$

$= \frac{x^{m+3n+4m-9n}}{x^{6m-6n}}$ $\left[\text{using } \frac{x^a \cdot x^b}{x^{a+b}}\right]$

$$= \frac{x^{5m-6n}}{x^{6m-6n}}$$

$$= x^{5m-6n-6m+6n} \quad \left[\text{using} \quad \frac{x^a}{x^b} = x^{a-b}\right]$$

$$= x^{-m}$$

2007 - May [13] If log (2a - 3b) = log a - log b, then a = :

(a) $\frac{3b^2}{2b-1}$ (b) $\frac{3b}{2b-1}$

(c) $\frac{b^2}{2b+1}$ (d) $\frac{3b^2}{2b+1}$ **(1 mark)**

Answer:

(a) Log (2a - 3b) = log a - log b

log (2a - 3b) = $\log\left(\frac{a}{b}\right)$

$2a - 3b = \frac{a}{b}$

$2ab - 3b^2 = a$

$2ab - a = 3b^2$

$a(2b - 1) = 3b^2$

$a = \frac{3b^2}{2b-1}$

2007 - Aug [14] On simplification $\frac{1}{1+z^{a-b}+z^{a-c}} + \frac{1}{1+z^{b-c}+z^{b-a}} + \frac{1}{1+z^{c-a}+z^{c-b}}$ reduces to :

(a) $\frac{1}{z^{2(a+b+c)}}$ (b) $\frac{1}{z^{(a+b+c)}}$

(c) 1 (d) 0 **(1 mark)**

Answer:

(c) $\frac{1}{1+z^{a-b}+z^{a-c}}$ $\frac{1}{1+z^{b-c}+z^{b-a}}$ $\frac{1}{1+z^{c-a}+z^{c-b}}$ +

$$= \frac{1}{1+\frac{z^{-b}}{z^{-a}}+\frac{z^{-c}}{z^{-a}}} + \frac{1}{1+\frac{z^{-c}}{z^{-b}}+\frac{z^{-a}}{z^{-b}}} + \frac{1}{1+\frac{z^{-a}}{z^{-b}}+\frac{z^{-b}}{z^{-c}}} +$$

$$= \frac{z^{-a}}{z^{-a}+z^{-b}+z^{-c}} + \frac{z^{-b}}{z^{-b}+z^{-c}+z^{-a}} + \frac{z^{-c}}{z^{-c}+z^{-a}+z^{-b}} +$$

$$= \frac{z^{-a}+z^{-b}+z^{-c}}{z^{-a}+z^{-b}+z^{-c}}$$

$= 1$

2007 - Aug [15] Ratio of earnings of A and B is 4 : 7. If the earnings of A increase by 50% and those of B decrease by 25%, the new ratio of their earning becomes 8 : 7. What is A's earning ?

(a) ₹ 21,000 (b) ₹ 26,000

(c) ₹ 28,000 (d) Data inadequate. **(1 mark)**

Answer:

(d) Let the earning of A and B be 4x and 7x respectively.

New earning of A = $4x \times 150\% = 6x$

New earning of B = $7x \times 75\% = 5.25$

Then, $\frac{6x}{5.25x} = \frac{8}{7}$

This does not give the value of x

So, the given data is inadequate.

2007 - Aug [16] P, Q and R are three cities. The ratio of average temperature between P and Q is 11 : 12 and that between P and R is 9 : 8. The ratio between the average temperature of Q and R is :

(a) 22 : 27 (b) 27 : 22

(c) 32 : 33 (d) None. **(1 mark)**

Answer:

(b) $\frac{P}{Q} = \frac{11}{12}$ and $\frac{P}{R} = \frac{9}{8}$

$\frac{P}{Q} = \frac{11\times9}{12\times9} = \frac{99}{108}$ and $\frac{P}{R} = \frac{9\times11}{8\times11} = \frac{99}{88}$

Therefore, $\frac{Q}{R} = \frac{108}{88} = \frac{27}{22}$

So, Q:R = 27:22

2007 - Aug [17] $\frac{1}{\log_{ab}(abc)} + \frac{1}{\log_{bc}(abc)} + \frac{1}{\log_{ca}(abc)}$ + is equal to :

(a) 0 (b) 1

(c) 2 (d) -1 **(1 mark)**

Answer:

(c) $\frac{1}{\log_{ab}^{(abc)}} + \frac{1}{\log_{bc}^{(abc)}} + \frac{1}{\log_{ca}^{(abc)}} +$

$= \frac{1}{\frac{\log(abc)}{\log(ab)}} + \frac{1}{\frac{\log(abc)}{\log(bc)}} + \frac{1}{\frac{\log(abc)}{\log(ca)}}$

$\left[\text{using } \log_a b = \frac{\log b}{\log a}\right]$

$= \frac{\log(ab)}{\log(abc)} + \frac{\log(bc)}{\log(abc)} + \frac{\log(ca)}{\log(abc)} +$

$= \frac{\log(ab \times bc \times ca)}{\log abc}$

$= \frac{\log a^2b^2c^2}{\log(abc)}$

$= \frac{\log(abc)^2}{\log abc} = \frac{2\log(abc)}{\log(abc)} = 2$

2007 - Aug [18] Number of digits in the numeral for 2^{64}. [Given log 2 = 0.30103] :

(a) 18 digits (b) 19 digits

(c) 20 digits (d) 21 digits. **(1 mark)**

Answer:

(c) 2^{64}

$= 64 \log 2$

$= 64 \times 0.30103$

$= 19.26592$

Number of digit in $2^{64} = 20$.

2007 - Nov [19] ₹ 407 are to be divided among A, B and C so that their shares are in the ratio $\frac{1}{4} \quad \frac{1}{5} \quad \frac{1}{6}$: . The respective shares of A, B, C are :

(a) ₹ 165, ₹ 132, ₹ 110 (b) ₹ 165, ₹ 110, ₹ 132
(c) ₹ 132, ₹ 110, ₹ 165 (d) ₹ 110, ₹ 132, ₹ 165 **(1 mark)**

Answer:

(a) The ratio of share of A, B and C

$= \frac{1}{4} \frac{1}{5} \frac{1}{6}$:

$= \frac{15:12:10}{60}$ = 15:12:10

Therefore, A's share = $407 \times \frac{15}{37}$ = ₹165

B's share = $407 \times \frac{12}{37}$ = ₹132

C's share = $407 \times \frac{10}{37}$ = ₹110

2007 - Nov [20] The incomes of A and B are in the ratio 3 : 2 and their expenditures in the ratio 5 : 3. If each saves ₹ 1,500, then B's income is :

(a) ₹ 6,000 (b) ₹ 4,500
(c) ₹ 3,000 (d) ₹ 7,500 **(1 mark)**

Answer:

(a) Let the income of A and B be 3*x* and 2*x* respectively and expenditures of A and B be 5y and 3y respectively.

Therefore, $3x - 5y = 1{,}500$ ..... (i)

$2x - 3y = 1{,}500$ ..... (ii)

Solving (i) and (ii) Simultaneously

We get $x = 3{,}000$ and $y = 1{,}500$

Therefore, B's income = $2x = 2 \times 3{,}000$ = ₹ 6,000

2007 - Nov [21] If $4^x = 5^y = 20^z$ then z is equal to :

(a) xy (b) $\frac{x+y}{xy}$

(c) $\frac{1}{xy}$ (d) $\frac{xy}{x+y}$ **(1 mark)**

Answer:

(d) $4^x = 5^y = 20^z = k$ (say)

$4 = k^{1/x}$

$5 = k^{1/y}$

$20 = k^{1/z}$

$4 \times 5 = 20$

$k^{1/x} \times k^{1/y} = k^{1/z}$

$k^{1/x + 1/y} = k^{1/z}$ ($x^m \times x^n = x^{m+n}$)

$k^{\frac{x+y}{xy}} = k^{1/z}$

Therefore, $\frac{x+y}{xy} = \frac{1}{z}$ ($x^m = x^n$ $m = n$)

$z = \frac{xy}{x+y}$

2007 - Nov [22] $\left(\frac{\sqrt{3}}{9}\right)^{5/2}\left(\frac{9}{3\sqrt{3}}\right)^{7/2} \times 9$ is equal to :

(a) 1 (b) $\sqrt{3}$

(c) $3\sqrt{3}$ (d) $\frac{3}{9\sqrt{3}}$ **(1 mark)**

Answer:

(a) $\left(\frac{\sqrt{3}}{9}\right)^{\frac{5}{2}}\left(\frac{9}{3\sqrt{3}}\right)^{\frac{7}{2}} \times 9$

$= \left(\frac{3^{\frac{1}{2}}}{3^2}\right)^{\frac{5}{2}}\left(\frac{3^2}{3.3^{\frac{1}{2}}}\right)^{\frac{7}{2}} \times 3^2$

$= \left(3^{\frac{1}{2}-2}\right)^{\frac{5}{2}}\left(\frac{3^2}{3^{\frac{3}{2}}}\right)^{\frac{7}{2}} \times 3^2$

$= \left(3^{\frac{-3}{2}}\right)^{\frac{5}{2}}\left(3^{\frac{2-3}{2}}\right)^{\frac{7}{2}} \times 3^2$

$= 3^{\frac{-15}{4}} \left(3^{\frac{1}{2}}\right)^{\frac{7}{2}} \times 3^2$

$3^{\frac{-15}{4}} \quad 3^{\frac{7}{4}} \times \quad \times 3^2$

$= 3^{\frac{-15}{4} + \frac{7}{4} + 2}$

$= 3^{-2+2} \quad = 3^0 = 1$

2007 - Nov [23] The value $\frac{\log_3 8}{\log_9 16 . \log_4 10}$ is :

(a) $3 \log_{10} 2$ (b) $7 \log_{10} 3$

(c) $3 \log_e z$ (d) None. **(1 mark)**

Answer:

(a) $\frac{\log_3^8}{\log_9^{16} - \text{Log}_4^{10}}$

$= \log_3^8 . \log_{16}^9 . \log_{10}^4$

$= \log_3^{2^3} . \text{Log}_{4^2}^{3^2} . \log_{10}^{2^3}$

$= 3\log_3^2 \frac{2}{4} \log_2^3 \quad 2 \log_{10}^2$

$= \frac{3\log 2}{\log 3} \quad \frac{1\log 3}{2\log 2} \quad \frac{2\log 2}{\log 10}$.

$= \frac{3\log 2}{\log 10}$

$= 3 \log_{10}^2$

2008 - Feb [24] In 40 litres mixture of glycerine and water, the ratio of glycerine and water is 3:1. The quantity of water added in the mixture in order to make this ratio 2:1 is:

(a) 15 litres (b) 10 litres

(c) 8 litres (d) 5 litres. **(1 mark)**

Answer:

(d) Quantity of glycerine = $40 \times \frac{3}{4}$ = 30 litres

Quantity of water = $40 \times \frac{1}{4}$ = 10 litres

Let x litres of water be added to the mixture.
Then, total quantity of mixture = (40 + x) litres
total quantity of water in the mixture = (10 + x) litres.

So, $\frac{30}{10+x} = \frac{2}{1}$

$30 = 20 + 2x$
$2x = 10$
$x = 5$ litres
Therefore, 5 litres of water must be added to the mixture.

2008 - Feb [25] The third proportional between $(a^2 - b^2)$ and $(a+b)^2$ is :

(a) $\frac{a+b}{a-b}$ (b) $\frac{a-b}{a+b}$

(c) $\frac{(a-b)^2}{a+b}$ (d) $\frac{(a+b)^3}{a-b}$ **(1 mark)**

Answer:

(d) Let the third proportional be x.

$\frac{a^2-b^2}{(a+b)^2} = \frac{(a+b)^2}{x}$

By cross – multiplication

$x = (a+b)^2 \frac{(a+b)^2}{(a^2-b^2)}$

$x = \frac{(a+b)^3}{(a-b)}$

2008 - Feb [26] If $2^X - 2^{X-1} = 4$ then x^x is equal to :

(a) 7 (b) 3

(c) 27 (d) 9 **(1 mark)**

Answer:

(c) $2^x - 2^{x-1} = 4$

$2^x - \frac{2^x}{2} = 4$

$2^x \left[1 - \frac{1}{2}\right] = 4$

$2^x \left[\frac{1}{2}\right] = 4$

$2^x = 8$
$2^x = 2^3$
$x = 3$
$x^x = 3^3$
$= 27$

2008 - Feb [27] If $x = \frac{e^n - e^{-n}}{e^n + e^{-n}}$, then the value of n is:

(a) $\frac{1}{2} \log \frac{1+x}{1-x}$ (b) $\log_e \frac{1+x}{1-x}$

(c) $\log_e \frac{1-x}{1+x}$ (d) $\log_e \frac{1-x}{1+x}$ **(1 mark)**

Answer:

(a) $x = \frac{e^n - e^{-n}}{e^n + e^{-n}}$

$\frac{1}{x} = \frac{e^n + e^{-n}}{e^n - e^{-n}}$

Applying Componendo & Dividendo

$\frac{1+x}{1-x} = \frac{e^n + e^{-n} + e^n - e^{-n}}{e^n + e^{-n} - e^n + e^{-n}}$

$\frac{1+x}{1-x} = \frac{2e^n}{2e^{-n}}$

$\frac{1+x}{1-x} = e^{2n}$ $= 2n$

$\text{Log}\left(\frac{1+x}{1-x}\right) = 2n$, $n = \frac{1}{2}\left(\text{Log}\frac{1+x}{1-x}\right)$ e

2008 - Feb [28] log 144 is equal to :

(a) 2 log 4 + 2 log 2 (b) 4 log 2 + 2 log 3

(c) 3 log 2 + 4 log 3 (d) 3 log 2 - 4 log 3 **(1 mark)**

Answer:

(b) Log 144

= Log (16 × 9) = log 16 + log9

= log 2^4 + log 3^2

= 4log2 + 2log3.

2008 - June [29] In what ratio should tea worth ₹ 10 per kg be mixed with tea worth ₹ 14 per kg, so that the average price of the mixture may be ₹ 11 per kg?

(a) 2:1 (b) 3:1

(c) 3:2 (d) 4:3 **(1 mark)**

Answer:

(b) Let x quantity of tea worth ₹10per kg. be mixed with y quantity worth 14 per kg.

Total price of the mixture $=10x+14y$.

and

Total quantity of the mixture $=x+y$

Average price of mixture will be $\frac{10x+14y}{x+y} = 11$

$10x + 14y = 11x + 11y$

$3y = x$

$\frac{x}{y} = \frac{3}{1}$

or $x : y = 3 : 1$ which is the required ratio.

2008 - June [30] The ages of two persons are in the ratio 5:7. Eighteen years ago their ages were in the ratio of 8:13, their present ages (in years) are :

(a) 50, 70 (b) 70, 50

(c) 40, 56 (d) None. **(1 mark)**

Answer:

(a) Let the present ages of persons be $5x$ & $7x$.

Eighteen years ago, their ages = $5x$ -18 and $7x$ - 18.

According to given:

$\frac{5x-18}{7x-18} = \frac{8}{13}$

$65x - 234 = 56x - 144$

$9x = 90$

$x = 10$

Their present ages are $5x = 5 \times 10 = 50$ years

$7x = 7 \times 10 = 70$ years.

2008 - June [31] If $x = y^a$, $y = z^b$ and $z = x^c$ then abc is:

(a) 2 (b) 1

(c) 3 (d) 4 **(1 mark)**

Answer:

(b) $Z = x^c$

$Z = (y^a)^c$ ($\because$ $y^a = x$)

$Z = y^{ac}$

$Z = (z^b)^{ac}$ ($\because$ $z^b = y$)

$Z = Z^{abc}$

$abc = 1$ ($\because$ $x^m = x^n$ then $m = n$)

2008 - June [32] If $\log_2 [\log_3 (\log_2 x)] = 1$, then x equals :

(a) 128 (b) 256

(c) 512 (d) None. **(1 mark)**

Answer:

(c) $\text{Log}_2 [\log_3(\log_2 x)] = 1$

$= \log_3 (\log_2 x) = 2^1$ (Converting into exponential form)

$= \log_2 x = 3^2$ (Converting into exponential form)

$= \log_2 x = 9$

$= x = 2^9$ (Converting into exponential form)

$x = 512$.

2008 - June [33] If $\log \left(\frac{a+b}{4}\right) = \frac{1}{2}$ (log a + log b) then:

$\frac{a}{b} + \frac{b}{a}$

(a) 12 (b) 14

(c) 16 (d) 8 **(1 mark)**

Answer:

(b) $\text{Log} \left(\frac{a+b}{4}\right) \quad \frac{1}{2} \quad =$ (Log a + Log b)

$\text{Log} \left(\frac{a+b}{4}\right)$ = log (ab)½

[Since, $\log_a mn = \log_a m + \log_a n$ and $n \log_a m = \log_a m^n$]

Take antilog on both sides.

$\frac{a+b}{4} \quad \sqrt{ab}$

$a + b = 4\sqrt{ab}$

Squaring both sides

$(a + b)^2 = (4\sqrt{ab})^2$

$a^2 + b^2 + 2ab = 16ab$

$a^2 + b^2 = 14ab$

$\frac{a}{b} + \frac{b}{a} = 14$, which is the required answer

2008 - June [34] If A, B and C started a business by investing ₹ 1,26,000, ₹ 84,000 and ₹ 2,10,000. If at the end of the year profit is ₹ 2,42,000 then the share of each is :

(a) 72,600, 48,400, 1,21,000 (b) 48,400, 1,21,000, 72,600

(c) 72,000, 49,000, 1,21,000 (d) 48,000, 1,21,400, 72,600

(1 mark)

Answer:

(a) Given : Capital invested by :

A : ₹ 126,000, B : ₹ 84,000, C: ₹ 2,10,000

The ratio of their investments is :

126 : 84 : 210 = 3 : 2 : 5

Profit (at year end) = ₹ 2,42,000 gives

A's Share $= \frac{3}{10} \times 2,42,000 =$ ₹ 72,600

B's Share $= \frac{2}{10} \times 2,42,000 =$ ₹ 48,400

C's Share $= \frac{5}{10} \times 2,42,000 =$ ₹ 1,21,000

2009 - June [35] If $\frac{p}{q} = -\frac{2}{3}$ then the value of $\frac{2p+q}{2p-q}$ is :

(a) 1 (b) -1/7

(c) 1/7 (d) 7 **(1 mark)**

Answer:

(c) $\frac{p}{q} = \frac{2}{3}$

So, $P = \frac{-2q}{3}$ .....(i)

Now, $\frac{2p + q}{2p - q}$

Substituting the value of p from (i)

$$\frac{2\left(\frac{-2q}{3}\right) + q}{2\left(\frac{-2q}{3}\right) - q}$$

$$= \frac{\frac{-4q}{3} + q}{\frac{-4q}{3} - q}$$

$$= \frac{\frac{-4q + 3q}{3}}{\frac{-4q - 3q}{3}}$$

$$= \frac{-q}{3} \times \frac{3}{-7q}$$

$$= \frac{1}{7}$$

2009 - June [36] Fourth proportional to x, 2x, (x+1) is:

(a) (x+2) (b) (x-2)

(c) (2x+2) (d) (2x-2) **(1 mark)**

Answer:

(c) Let the fourth proportional to x, 2x, (x + 1) be t, then,

$$\frac{x}{2x} = \frac{x + 1}{t}$$

$$\frac{1}{2} = \frac{x + 1}{t}$$

t = 2x + 2

∴ Fourth proportional to x, 2x, (x + 1) is (2x + 2)

i.e. x: 2x :: (x + 1) : (2x +2)

2009 - June [37] If $x = 3^{1/3} + 3^{-1/3}$ then find the value of $3x^3 - 9x$

(a) 3 (b) 9

(c) 12 (d) 10 **(1 mark)**

Answer:

(d) $x = 3^{1/3} + 3^{-1/3}$ ..... (1)

On cubing both sides, we get

$x^3 = (3^{1/3} + 3^{-1/3})^3$

$x^3 = 3 + 3^{-1} + 3 \times 3^{1/3} \times \frac{1}{3^{1/3}} \quad (3^{1/3} + 3^{-1/3})$

$x^3 = 3 + \frac{1}{3} + 3(3^{1/3} + 3^{-1/3})$

$x^3 = 3 + \frac{1}{3} + 3x$ [Using (1)]

$x^3 - 3x = \frac{9+1}{3}$

$3(x^3 - 3x) = 10$

$3x^3 - 9x = 10$

2009 - June [38] Find the value of :

$[1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$

(a) 1/x (b) x

(c) 1 (d) None of these. **(1 mark)**

Answer:

(b) $[1 - \{1 - (1 - x^2)^{-1}\}^{-1}]^{-1/2}$

$= \left[1 - \left\{1 - \frac{1}{1 - x^2}\right\}^{-1}\right]^{-1/2}$

$= \left[1 - \left\{\frac{1 - x^2 - 1}{1 - x^2}\right\}^{-1}\right]^{-1/2}$

$$= \left[1 - \left\{\frac{-x^2}{1-x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1 - \left\{\frac{1-x^2}{x^2}\right\}^{-1}\right]^{-1/2}$$

$$= \left[1 + \frac{1-x^2}{x^2}\right]^{-1/2} \qquad \left[\frac{x^2+1-x^2}{x^2}\right]^{-1/2}$$

$$= \left[\frac{1}{x^2}\right]^{-1/2} \qquad = (x^2)^{1/2}$$

$$= x$$

2009 - June [39] log (m + n) = log m + log n, m can be expressed as :

(a) $m = \frac{n}{n-1}$ (b) $m = \frac{n}{n+1}$

(c) $m = \frac{n+1}{n}$ (d) $m = \frac{n+1}{n-1}$ **(1 mark)**

Answer:

(a) log (m + n) = log m + log n

log (m + n) = log (m n) [∵ log (ab) = log a + log b]

Taking Antilog on both side

Antilog [log (m + n)] = Antilog [log mn]

∴ m + n = mn

mn – m = n

m (n - 1) = n

$m = \frac{n}{n-1}$

2009 - June [40] $\log_4 (x^2 + x) - \log_4 (x+1) = 2$. Find x

(a) 16 (b) 0

(c) – 1 (d) None of these. **(1 mark)**

Answer:

(a) $\text{Log}_4 (x^2 + x) - \text{Log}_4 (x+1) = 2$

$\text{Log}_4 \left(\frac{x^2+x}{x+1}\right) \qquad \left[\because \log_a m - \log_a n = \log_a \left(\frac{m}{n}\right)\right]$

$4^2 = \frac{x^2+x}{x+1}$

$$16 = \frac{x^2 + x}{x + 1}$$

$16x + 16 = x^2 + x$
$x^2 - 15x - 16 = 0$
$x^2 - 16x + x - 16 = 0$
$x(x - 16) + 1(x - 16) = 0$
$(x + 1)(x - 16) = 0$
$x = -1$ or $x = 16$
Since $x = -1$ is not possible therefore $x = 16$

2009 - Dec [41] $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

(a) 1/2 (b) 3/2
(c) 2/3 (d) 1/3 **(1 mark)**

Answer:

(b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$

$= 2^n\left(1 + \frac{1}{2}\right)$

$2n(2 - 1)$

$= \frac{3}{2} \quad \frac{3}{2}$

$= 1$

2009 - Dec [42] If $2^x \times 3^y \times 5^z = 360$. Then what is the value of x, y, z.?

(a) 3, 2, 1 (b) 1, 2, 3
(c) 2, 3, 1 (d) 1, 3, 2 **(1 mark)**

Answer:

(a) $2^x \times 3^y \times 5^z = 360$. .....(1)
The factors of 360 are:-
$2^3 \times 3^2 \times 5$.
$2^3 \times 3^2 \times 5^1 = 360$.....(2)
On comparing (1) and (2), we get;
$x = 3$, $y = 2$ and $z = 1$

2009 - Dec [43] Find the value of $\left[\log_{10}\sqrt{25} - \log_{10}(2^3) + \log_{10}(4)^2\right]^x$

(a) x (b) 10

(c) 1 (d) None. **(1 mark)**

Answer:

(c) $\left[\log_{10}\sqrt{25} - \log_{10}(2^3) + \log_{10}(4^2)\right]^x$

$= [\log_{10} 5 - 3\log_{10}2 + \log_{10}(2^4)]^x$

$= [\log_{10} 5 - 3\log_{10}2 + 4\log_{10}{}^2]^x$

$= [\log_{10} 5 + \log_{10}{}^2]^x$

$= [\log_{10}(5 \times 2)]^x$ [$\because$ log (mn) = log m + log n]

$= [\log_{10} 10]^x$

$= 1^x$ [$\because$ $\log_a a = 1$]

$= 1$

2010 - June [44] If $2^X - 2^{X-1} = 4$ then x^x is equal to :

(a) 7 (b) 3

(c) 27 (d) 9 **(1 mark)**

Answer:

Please refer 2008 - Feb [26] on page no. 22

2010 - June [45] If $\log_a b + \log_a c = 0$ then

(a) b = c (b) b = - c

(c) b = c = 1 (d) b and c are reciprocals. **(1 mark)**

Answer:

(d) $\log_a b \quad \log_a c \quad = 0$

$\log_a bc \quad = 0$

$a^0 = bc$

$bc = 1$

$\therefore \quad b = \frac{1}{c}$

So, b and c are reciprocals.

2010 - June [46] What must be added to each term of the ratio 49 : 68, so that it becomes 3 : 4 ?

(a) 3 (b) 5

(c) 8 (d) 9 **(1 mark)**

Answer:

(c) Let the number added be x

$$\frac{49 + x}{68 + x} = \frac{3}{4}$$

$196 + 4x = 204 + 3x$

$x = 8$

2010 - June [47] The students of two classes are in the ratio 5 : 7, if 10 students left from each class, the remaining students are in the ratio of 4 : 6 then the number of students in each class is:

(a) 30, 40 (b) 25, 24

(c) 40, 60 (d) 50, 70 **(1 mark)**

Answer:

(d) Let the ratio be 5x : 7x

If 10 student left, Ratio became 4 : 6

$$\frac{5x - 10}{7x - 10} = \frac{4}{6}$$

$30x - 60 = 28x - 40$

$2x = 20$

$x = 10$

$\therefore$ No. of students in each class is 5x and 7x

i.e. 50, 70

2010 – Dec [48] The value of $2 \log x + 2 \log x^2 + 2 \log x^3 + \text{-------} + 2 \log x^n$ will be :

(a) $\frac{n(n+1) \log x}{2}$ (b) n(n+1) log x

(c) $n^2 \log x$ (d) None of these. **(1 mark)**

Answer:

(b) $2 \log x + 2 \log x^2 + 2\log x^3 + ...$

$2[\log x + \log x^2 + \log x^3 + ...]$

$2[\log x + 2\log x + 3\log x + ...]$

$2\log x[1 + 2 + 3 ..... n]$

$2 \log x \times \frac{n(n+1)}{2}$

$= n(n + 1) \log x$

2010 – Dec [49] The recurring decimal 2.7777..... can be expressed as:

(a) 24/9 (b) 22/9

(c) 26/9 (d) 25/9 **(1 mark)**

Answer:

(d) 2.7777

$2 + 0.7 + 0.07 + 0.007 + ...$

$2 + \left(\frac{7}{10} + \frac{7}{100} + \frac{7}{1000} + ...\right)$

$2 + 7\left(\frac{1}{10} + \frac{1}{100} + \frac{1}{1000} + ...\right)$

$2 + 7\left(\frac{1/10}{1 - 1/10}\right)$

$= 2 + 7 \times \frac{1}{9}$

$= 2 + \frac{7}{9}$

$= \frac{18+7}{9}$

$= \frac{25}{9}$

2010 – Dec [50] Solve : $\left(\frac{\log x_{10} - 3}{2}\right) + \left(\frac{11 - \log x_{10}}{3}\right) = 2$

(a) 10^{-1} (b) 10^{2}

(c) 10 (d) 10^{3} **(1 mark)**

Answer:

(a) $\left(\frac{\log_{10}x - 3}{2}\right) \quad \left(\frac{11 - \log_{10}x}{3}\right) = 2$

$3 \log_{10}x - 9 + 22 - 2 \log_{10} x = 12$

$\log_{10}x + 13 = 12$

$\log_{10}x = -1$

$x = 10^{-1}$

2010 – Dec [51] If A:B= 2:5, then (10A + 3B):(5A+2B) is equal to:

(a) 7:4 (b) 7:3

(c) 6:5 (d) 7:9 **(1 mark)**

Answer:

(a) $\frac{A}{B} \quad \frac{2}{5} \quad \frac{2k}{5k}$

$\frac{10A + 3B}{5A + 2B} \quad \frac{20k + 15k}{10k + 10k} \quad \frac{35k}{20k} =$

$= \frac{35}{20}$

$= \frac{7}{4}$

2011 – June [52] If n = m! where ('m' is a positive integer > 2) then the value of :

$$\frac{1}{\log_2^n} + \frac{1}{\log_3^n} + \frac{1}{\log_4^n} + \ldots\ldots\ldots + \frac{1}{\log_m^n}$$

(a) 1 (b) 0

(c) -1 (d) 2 **(1 mark)**

Answer:

(a) Given : n = M ! for M ≥ 2

$\frac{1}{\log_2^n} \quad \frac{1}{\log_3^n} \quad \frac{1}{\log_4^n} \quad + \quad \frac{1}{\log_m^n} + \ldots\ldots\ldots\ldots +$

or, $= \log_n^2 \quad \log_n^3 \quad \log_n^4 \quad + \quad \log_n^m \ldots\ldots\ldots\ldots + \quad \left(\therefore \log_b^a = \frac{1}{\log_a^b}\right)$

$= \log_n \quad (2 \times 3 \times 4 \times \ldots\ldots\ldots \times m) \quad \left(\therefore \log^{(mn)} = \log^m + \log^n\right)$

$= \log_n \quad (m!)$

$= \log_n^n$

$= 1$

2011 – June [53] In a film shooting, A and B received money in a certain ratio and B and C also received the money in the same ratio. If A gets ₹ 1,60,000 and C gets ₹ 2,50,000. Find the amount received by B ?

(a) ₹ 2,00,000 (b) ₹ 2,50,000

(c) ₹ 1,00,000 (d) ₹ 1,50,000 **(1 mark)**

Answer:

(a) Given : A : B = B : C

$\Rightarrow B^2 = A \times C$

or $B = \sqrt{A \times C}$

& A = 1,60,000 ; C = 2,50,000

$\because B = \sqrt{1,60,000 \times 2,50,000}$

B = 2,00,000

2011 – Dec [54] The ratio Compounded of 4:5 and sub-duplicate of "a":9 is 8:15. Then Value of "a" is:

(a) 2 (b) 3

(c) 4 (d) 5 **(1 mark)**

Answer:

(c) Sub duplicate ratio of a : 9 = $\sqrt{a}$ $\sqrt{9}$, Compound Ratio (C.R.) = 8:15

Compound Ratio of 4:5 and sub duplicate ratio of a:9 is given by

$C.R = \frac{4}{5} \times \frac{\sqrt{a}}{\sqrt{9}}$

$\frac{8}{15} \quad \frac{4}{5} = \frac{\sqrt{a}}{\sqrt{9}} \times$

$\sqrt{a} \quad \frac{8 \times 5 \times \sqrt{9}}{15 \times 4}$

$\sqrt{a} \quad \frac{8 \times 5 \times 3}{15 \times 4}$

$\sqrt{a} \quad = 2$

On squaring $(\sqrt{a})^2 \quad = 2^2$

a = 4

2011 – Dec [55] If $\log_2 x + \log_4 x = 6$, then the Value of x is :

(a) 16 (b) 32

(c) 64 (d) 128 **(1 mark)**

Answer:

(a) If $\log_2 x + \log_4 x = 6$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 4} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{\log 2^2} = 6$$

$$\frac{\log x}{\log 2} + \frac{\log x}{2\log 2} = 6$$

$$\frac{\log x}{\log 2}\left[1 + \frac{1}{2}\right] = 6$$

$$\frac{\log x}{\log 2} \cdot \frac{3}{2} = 6$$

$$\frac{\log x}{\log 2} = 6 \times \frac{2}{3}$$

$$\frac{\log x}{\log 2} = 4$$

$\log x = 4 \log 2$

$\log x = \log 2^4$

$x = 2^4$

$x = 16$

2011 – Dec [56] If X Varies inversely as square of Y and given that Y = 2 for X = 1, then the Value of X for Y = 6 will be:

(a) 3 (b) 9

(c) 1/3 (d) 1/9 **(1 mark)**

Answer:

(d) Given x varies inversely as square of y

i. e. $x \propto \frac{1}{y^2}$

$x = k \frac{1}{y^2}$

$x = \frac{k}{y^2}$ .....(1)

Given x = 1, y = 2 then

$1 = \frac{k}{(2)^2} \Rightarrow$ $k = 1 \times 4 = 4$

Now putting $y = 6$, $k = 4$ in equation (1)

$x = \frac{4}{6^2}$

$x = \frac{4}{36} = \frac{1}{9}$

2012 - June [57] The value of $\frac{(3^{n+1} + 3^n)}{(3^{n+3} - 3^{n+1})}$ is equal to:

(a) 1/5 (b) 1/6

(c) 1/4 (d) 1/9 **(1 mark)**

Answer:

(b) $\frac{3^{n+1} + 3^n}{3^{n+3} - 3^{n+1}}$

$= \frac{3^n . 3^1 + 3^n}{3^n . 3^3 - 3^n . 3^1}$

$= \frac{3^n (3^1 + 1)}{3^n (3^3 - 3)}$

$= \frac{(3 + 1)}{(27 - 3)}$

$= \frac{4}{24}$

$= \frac{1}{6}$

2012 - June [58] If $\log_x y = 100$ and $\log_2 x = 10$, then the value of 'y' is :

(a) 2^{10} (b) 2^{100}

(c) $2^{1,000}$ (d) $2^{10,000}$ **(1 mark)**

Answer:

(c) Given

$\log_x y = 100$ .....(1)

$\log_2 x = 10$.....(2)

Multiply eq (1) & (2)

$\log_x y \cdot \log_2 x = 100 \times 10$

$\frac{\log y}{\log x} \frac{\log x}{\log 2} = 1{,}000$

$\log y = 1{,}000 \log 2$

$\log y = \log 2^{1,000}$

$\Rightarrow \quad y = 2^{1,000}$

2012 - June [59] Which of the numbers are not in proportion ?

(a) 6, 8, 5, 7 (b) 7, 14, 6

(c) 18, 27, 12, 18 (d) 8, 6, 12, 9 **(1 mark)**

Answer:

(a) If say a, b, c, d are in proportion they bear a common ratio that is

$\Rightarrow \frac{a}{b} \quad \frac{c}{d} =$

Option (A) $\frac{6}{8} \neq \frac{5}{7}$

Option (B) $\frac{7}{3} = \frac{14}{6}$

Option (C) $\frac{18}{27} = \frac{12}{18}$

Option (D) $\frac{8}{6} = \frac{12}{9}$

2012 - Dec [60] Find the value of x, if $x (x)^{⅓} = (x^{⅓})^x$

(a) 3 (b) 4

(c) 2 (d) 6 **(1 mark)**

Answer:

(b) If $x^1 (x)^{1/3} = (x^{1/3})^x$

$x^{1+1/3} = x^{\frac{1}{3}x}$

$x^{4/3} = x^{\frac{1}{3}x}$

on comparing

$\frac{4}{3} \times \frac{x}{3}$

$3x = 12 \quad x = 4$

2012 - Dec [61] Which of the following is true.

If $\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} + \frac{1}{abc} =$

(a) $\log (ab + bc + ca) = abc$ (b) $\log \left(\frac{1}{a} + \frac{1}{b} + \frac{1}{c} \right) = abc$

(c) $\log (abc) = 0$ (d) $\log (a + b + c) = 0$ **(1 mark)**

Answer:

(d) Given

$\frac{1}{ab} + \frac{1}{bc} + \frac{1}{ca} = \frac{1}{abc}$

$\frac{c + a + b}{abc} = \frac{1}{abc}$

$a + b + c = 1$

taking log on both side

$\log (a + b + c) = \log 1$

$\log (a + b + c) = 0$

2012 - Dec [62] Find two numbers such that mean proportional between them is 18 and third proportional between them is 144

(a) 9, 36 (b) 8, 32

(c) 7, 28 (d) 6, 24 **(1 mark)**

Answer:

(a) Let two Nos. be x and y

Mean proportion between x and y is 18

So, x, 18, y are in proportion

$x : 18 :: 18 : y$

$\frac{x}{18} = \frac{18}{y}$

$xy = 324$

$x = \frac{324}{y}$ _____ (1)

If third proportion between x & y be 144

So, x, y, 144 are in proportion

$x : y :: y : 144$

$\frac{x}{y} = \frac{y}{144}$

$y^2 = 144x$ _____ (2)

Putting the value of x in equation (2)

$y^2 = 144 \times \frac{324}{y}$

$y^3 = 144 \times 324$

$y = \sqrt[3]{144 \times 324}$

$y = \sqrt[3]{3 \times 3 \times 2 \times 2 \times 2 \times 2 \times 2 \times 2 \times 3 \times 3 \times 3 \times 3}$

$y = \sqrt[3]{6 \times 6 \times 6 \times 6 \times 6 \times 6}$

$y = 6 \times 6$

$y = 36$

Putting y = 36 in equation (1)

$x = \frac{324}{36} = 9$

x = 9, y = 36

2013 - June [63] For what value of x, the equation $(\log_{\sqrt{x}} 2)^2 = \log_x^2$ is true?

(a) 16 (b) 32

(c) 8 (d) 4 **(1 mark)**

Answer:

(a) Given

$\left(\log_{\sqrt{x}} 2\right)^2 = \log_x 2$

$\left(\frac{\log 2}{\log \sqrt{x}}\right)^2 = \left(\frac{\log 2}{\log x}\right)$

$\left(\frac{\log 2}{\log x^{1/2}}\right)^2 = \frac{\log 2}{\log x}$

$\left(\frac{\log 2}{\frac{1}{2}\log x}\right)^2 = \frac{\log 2}{\log x}$

$\left(\frac{2\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)$

$4\left(\frac{\log 2}{\log x}\right)^2 = \left(\frac{\log 2}{\log x}\right)^1$

$4\ \frac{\log 2}{\log x} = 1$

$4 \log 2 = \log x$

$\log 2^4 = \log x$

$\Rightarrow \quad 2^4 = \Rightarrow \boxed{x} = 16$

2013 - June [64] The mean proportional between 24 and 54 is :

(a) 33 (b) 34

(c) 35 (d) 36 **(1 mark)**

Answer:

(d) Mean Proportion $= \sqrt{24 \times 54}$

$= \sqrt{1296}$

$= 36$

2013 - June [65] The triplicate ratio of 4 : 5 is:

(a) 125 : 64 (b) 16 : 25

(c) 64 : 125 (d) 120 : 46 **(1 mark)**

Answer:

(c) The triplicate Ratio of $4 : 5 = 4^3 : 5^3$

$= 64 : 125$

2013 - Dec [66] If $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$ = 0, then $\left(\frac{a+b+c}{3}\right)^3$ of

(a) abc (b) 9abc

(c) $\frac{1}{abc}$ (d) $\frac{1}{9abc}$ **(1 mark)**

Answer:

(a) If $\sqrt[3]{a}+\sqrt[3]{b}+\sqrt[3]{c}$ = 0

$a^{1/3} + b^{1/3} + c^{1/3} = 0$

$a^{1/3} + b^{1/3} = - c^{1/3}$ ..... (i)

Cube on both side

$(a^{1/3} + b^{1/3})^3 = (- c^{1/3})^3$

$(a^{1/3})^3 + (b^{1/3})^3 + 3 . a^{1/3}. b^{1/3} (a^{1/3} + b^{1/3}) = - c$

$a + b + 3a^{1/3}. b^{1/3}. (- c^{1/3}) = - c$

$a + b - c\ 3a^{1/3}. b^{1/3}. c^{1/3} = - c$

$a + b + c = 3a^{1/3}. b^{1/3}. c^{1/3}$

$\left(\frac{a+b+c}{3}\right) \quad \frac{3a^{1/3}.b^{1/3}.c^{1/3}}{3}$

$\left(\frac{a+b+c}{3}\right)^3 = (a^{1/3}. b^{1/3}. c^{1/3})^3 = abc$

2013 - Dec [67] Find three numbers in the ratio 1 : 2 : 3, so that the sum of their squares is equal to 504

(a) 6, 12, 18 (b) 3, 6, 9

(c) 4, 8, 12 (d) 5, 10, 15 **(1 mark)**

Answer:

(a) Since Ratio of three Number is 1 : 2 : 3

First No. = x

Second No. = 2x

Third No. = 3x

Sum of squares of numbers = 504

$(x)^2 + (2x)^2 + (3x)^2 = 504$

$x^2 + 4x^2 + 9x^2 = 504$

$14x^2 = 504$

$x^2 = \frac{504}{14}$

$x^2 = 36$
$x = 6$
First No. = $x = 6$
Second No. = $2x = 2 \times 6 = 12$
Third No. = $3x = 3 \times 6 = 18$

2013 - Dec [68] The value of $\log_4 9 \cdot \log_3 2$ is:
(a) 3 (b) 9
(c) 2 (d) 1 **(1 mark)**
Answer:
(d) $\log_4 9 \cdot \log_3 2$

$$= \frac{\log 9}{\log 4} \cdot \frac{\log 2}{\log 3}$$

$$= \frac{\log 3^2}{\log 2^2} \cdot \frac{\log 2}{\log 3}$$

$$= \frac{2\log 3}{2\log 2} \cdot \frac{\log 2}{\log 3}$$

$= 1$

2013 - Dec [69] The value of $(\log_y x \cdot \log_z y \cdot \log_x z)^3$ is
(a) 0 (b) - 1
(c) 1 (d) 3 **(1 mark)**
Answer:
(c) $(\log_y x \cdot \log_z y \cdot \log_x z)^3$

$$= \left(\frac{\log x}{\log y} \cdot \frac{\log y}{\log z} \cdot \frac{\log z}{\log x}\right)^3$$

$= (1)^3$
$= 1$

2013 - Dec [70] Divide 80 into two parts so that their product is maximum, then the numbers are:
(a) 25, 55 (b) 35, 45
(c) 40, 40 (d) 15, 65 **(1 mark)**
Answer:
(c) The sum of two No. = 80
First No. = x
Second No. = (80 - x)

Product two No = x. (80 - x)

$P = 80x - x^2$ ..... (1)

w.r.f. (x)

$\frac{dp}{dx} = 80 - 2x$ ..... (2)

$\frac{d^2p}{dx^2} = -2$ ..... (3)

For max/minima

$\frac{dp}{dx} = 0$

$80 - 2x = 0$

$2x = 80$

$x = 40$

$x = 40$ in equation (iii)

$\frac{d^2p}{dx^2} = -2$ (Negative)

function is maximum at x = 40

Numbers are 40, (80 - 40)

= 40, 40

2014 - June [71] If x : y = 2:3, then (5x+2y):(3x-y)= ___

(a) 19 : 3 (b) 16 : 3

(c) 7 : 2 (d) 7 : 3 **(1 mark)**

Answer:

(b) Given,

x : y = 2 : 3

Let x = 2k, y = 3k

(5x + 2y) : (3x - y)

$= \frac{(5x + 2y)}{(3x - y)}$

$= \frac{5 \times 2k + 2 \times 3k}{3 \times 2k - 3k}$

$= \frac{10k + 6k}{6k - 3k}$

$= \frac{16k}{3k}$

= 16 : 3

2014 - June [72] If $(25)^{150} = (25x)^{50}$; then the value of x will be :

(a) 5^3 (b) 5^4

(c) 5^2 (d) 5 **(1 mark)**

Answer:

(b) If $(25)^{150} = (25x)^{50}$

$25^{150} = 25^{50} . x^{50}$

$\frac{25^{150}}{25^{50}} = x^{50}$

$25^{100} = x^{50}$

$(5^2)^{100} = x^{50}$

$5^{200} = x^{50}$

$(5^4)^{50} = x^{50}$

$5^4 = x$

$x = 5^4$

2014 - June [73] The value of $\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \times \left(\frac{y^b}{y^c}\right)^{b^2+bc+c^2} \times \left(\frac{y^c}{y^a}\right)^{c^2+ac+a^2}$ to ___ .

(a) y (b) - 1

(c) 1 (d) None of these **(1 mark)**

Answer:

(c) $\left(\frac{y^a}{y^b}\right)^{a^2+ab+b^2} \left(\frac{y^b}{y^c}\right)^{b^2+bc+c^2} \left(\frac{y^c}{y^a}\right)^{c^2+ac+a^2}$.

$= (y^{a-b})^{a^2+ab+b^2} \; (y^{b-c})^{b^2+bc+c^2} . \; (y^{c-a})^{c^2+ac+a^2}$.

$= y^{a^3-b^3} \; y^{b^3-c^3} . \; y^{c^3-a^3}$.

$= y^{a^3-b^3+b^3-c^3+c^3-a^3}$

$= y^0 = 1$

2014 - June [74] If the salary of P is 25% lower than that of Q and the salary of R is 20% higher than that of Q, the ratio of the salary of R and P will be:

(a) 5 : 8 (b) 8 : 5

(c) 5 : 3 (d) 3 : 5 **(1 mark)**

Answer:

(b) Let Salary of Q = 100

Salary of P = 100 - 25% of 100
= 100 - 25
= 75

Salary of R = 100 + 20% of 100
= 100 + 20
= 120

Ratio of salary of R and P = 120 : 75 = 8 : 5

2014 - June [75] If $x^2+y^2=7xy$, then $\log \frac{1}{3}$ (x+y) = ___.

(a) (log x+log y) (b) $\frac{1}{2}$ (log x + log y)

(c) $\frac{1}{3}$ (log x / log y) (d) $\frac{1}{3}$ (log x + log y) **(1 mark)**

Answer:

(b) If $x^2 + y^2 = 7xy$

$x^2 + y^2 + 2xy = 7xy + 2xy$

$(x + y)^2 = 9xy$

taking log on both side

$\log (x + y)^2 = \log 9xy$

$2 \log (x + y) = \log 9 + \log x + \log y$

$2 \log (x + y) = \log 3^2 + \log x + \log y$

$2 \log (x + y) = 2 \log 3 + \log x + \log y$

$2 \log (x + y) - 2 \log 3 = \log x + \log y$

$2 \left[\log \frac{(x + y)}{3}\right] = \log x + \log y$

$\log \frac{(x + y)}{3} \quad \frac{1}{2} = [\log x + \log y]$

2014 - June [76] A person has assets worth ₹ 1,48,200. He wish to divide it amongst his wife, son and daughter in the ratio 3 : 2 : 1 respectively. From this assets, the share of his son will be:

(a) ₹ 24,700 (b) ₹ 49,400

(c) ₹ 74,100 (d) ₹ 37,050 **(1 mark)**

Answer:

(b) A person has Assets worth = ₹ 1,48,200

Ratio of share of wife, son & daughter

= 3 : 2 : 1

Sum of Ratio = 3 + 2 + 1 = 6

Share of Son = $\frac{2}{6} \times 1,48,200$

= 49,400

2014 - June [77] If $x = \log_{24}12$, $y = \log_{36}24$ and $z = \log_{48}36$, then xyz + 1 = ____

(a) 2xy (b) 2xz

(c) 2yz (d) 2 **(1 mark)**

Answer:

(c) If $x = \log_{24}12$, $y = \log_{36}24$ and $z = \log_{48}36$ then

XYZ + 1

$= \log_{24}12 \times \log_{36}24 \times \log_{48}36 + 1$

$= \frac{\log 12}{\log 24} \cdot \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48} + 1$

$= \frac{\log 12}{\log 48} + 1$

$= \frac{\log 12 + \log 48}{\log 48}$

$= \frac{\log(12 \times 48)}{\log 48}$

$= \frac{\log(576)}{\log 48}$

$= \frac{\log 24^2}{\log 48}$

$= \frac{2\log 24}{\log 48}$

$= 2 . \frac{\log 24}{\log 36} \cdot \frac{\log 36}{\log 48}$

$= 2.\log_{36}24 . \log_{48}36$

= 2 y z

2014 - Dec [78] If log x = a + b, log y = a - b then the value of log $\frac{10x}{y^2}$ = _____.

(a) 1 - a + 3b (b) a - 1 + 3b
(c) a + 3b + 1 (d) 1 - b + 3a **(1 mark)**

Answer:

(a) Given log x = a + b, log y = a - b

$\log\left(\frac{10x}{y^2}\right)$ = log 10x - log y^2

= log 10 + log x - 2log y
= 1 + (a + b) - 2 (a - b)
= 1 + a + b - 2a + 2b
= 1 - a + 3b

2014 - Dec [79] If $x = 1 + \log_p qr$, $y = 1 + \log_q rp$ and $z = 1 + \log_r pq$ then the value of $\frac{1}{x} + \frac{1}{y} + \frac{1}{z}$ = _____

(a) 0 (b) 1
(c) -1 (d) 3 **(1 mark)**

Answer:

(b) If $x = 1 + \log_p qr$, $y = 1 + \log_q rp$, $z = 1 + \log_r pq$

$x = 1 + \frac{\log qr}{\log p}$

$x = \frac{\log p + \log qr}{\log p}$

$x = \frac{\log pqr}{\log p}$

$\frac{1}{x} = \frac{\log p}{\log pqr}$

Similarly

$\frac{1}{y} = \frac{\log q}{\log pqr}$

$\frac{1}{z} = \frac{\log r}{\log pqr}$

$\frac{1}{x} + \frac{1}{y} + \frac{1}{z} = \frac{\log p}{\log pqr} + \frac{\log q}{\log pqr} + \frac{\log r}{\log pqr}$ +

$$= \frac{\log p + \log q + \log r}{\log pqr}$$
$$= \frac{\log pqr}{\log pqr}$$
$$= 1$$

2014 - Dec [80] For three months, the salary of a person are in the ratio 2 : 4 : 5. If the difference between the product of salaries of the first two months and last two months is ₹ 4,80,00,000; then the salary of the person for the second month will be:

(a) ₹ 4,000 (b) ₹ 6,000
(c) ₹ 8,000 (d) ₹ 12,000 **(1 mark)**

Answer:

(c) Ratio of the salary of a person in three months = 2 : 4 : 5

Let, Salary of Ist month = 2x
Salary of IInd month = 4x
Salary of IIIrd month = 5x

Given

(Salary of Product of last two months) - (Salary of Product Ist two months)
= 4,80,00,000
$(4x.5x) - (2x.4x)$ = 4,80,00,000
$20x^2 - 8x^2$ = 4,80,00,000
$12x^2$ = 4,80,00,000
x^2 = 40,00,000
x = 2,000

Salary of the person for second month = 4x = 4 × 2,000 = 8,000

2015 - June [81] A dealer mixes rice costing ₹ 13.84 per Kg. with rice costing ₹ 15.54 and sells the mixture at ₹ 17.60 per Kg. So, he earns a profit of 14.6% on his sale price. The proportion in which he mixes the two qualities of rice is:

(a) 3 : 7 (b) 5 : 7
(c) 7 : 9 (d) 9 : 11 **(1 mark)**

Answer:

(a) Let SP of mixture is ₹ 100

Then Profit = 14.6% of 100
= 14.6

CP of mixture = (100 - 14.6)
= 85.4

∵ If SP is ₹ 100 then CP = 85.4

∴ If SP is ₹ 1 then CP = $\frac{85.4}{100}$

∴ If SP is ₹ 17.60 then CP = $\frac{85.4}{100} \times 17.60$

= 15.0304

∴ CP of the Mixture per kg = ₹ 15.0304

2^{nd} difference = Profit by SP 1 kg of 2^{nd} kind @ ₹ 15.0304
= 15.54 - 15.0304
= 0.5096

1^{st} difference = ₹ 15.0304 - 13.84
= ₹ 1.1904

The Require Ratio = (2^{nd} difference) : (1^{st} difference)
= 0.5096 : 1.1904
= 3 : 7

2015 - June [82] If $p^x = q$, $q^y = r$ and $r^2 = p^6$, then the value of xyz will be:

(a) 0 (b) 1
(c) 3 (d) 6 **(1 mark)**

Answer:

(d) If $p^x = q$, $q^y = r$ and $r^z = p^6$

$q = p^x$, $q^y = r$ and $r^z = p^6$

$(q^y)^z = p^6$

$[(p^x)^y]^z = p^6$

$p^{xyz} = p^6 = xyz = 6$

2015 - June [83] If log x = m + n and log y = m - n, then log $(10x/y^2)$ =

(a) 3n - m + 1 (b) 3m - n + 1
(c) 3n + n + 1 (d) 3m + n + 1 **(1 mark)**

Answer:

(a) Log x = m + n and log y = m - n

Then $\log\left(\frac{10x}{y^2}\right)$ = log 10x - log y^2

= log 10 + log x - 2 log y

= 1 + log x - 2 log y

= 1 + (m + n) - 2 (m - n)

= 1 + m + n - 2m + 2n

= 3n - m + 1

2015 - June [84] If $15(2p^2 - q^2) = 7pq$, where p and q are positive, then p : q will be:

(a) 5 : 6 (b) 5 : 7

(c) 3 : 5 (d) 8 : 3 **(1 mark)**

Answer:

(a) If $15(2p^2 - q^2) = 7pq$

$30p^2 - 15q^2 = 7pq$

$30p^2 - 7pq - 15q^2 = 0$

$30p^2 - 25pq + 18pq - 15q^2 = 0$

$5p(6p - 5q) + 3q(6p - 5q) = 0$

$(6p - 5q)(5p + 3q) = 0$

If $6p - 5q = 0$ and $5p + 3q = 0$

$6p = 5q$ $5p = -3q$

$\frac{p}{q} \quad \frac{5}{6}$ $= p : q = \frac{p}{q} : 6\frac{-3}{5} =$

(not possible)

2015 - Dec [85] The ratio of third proportion of 12, 30 to the mean proportion of 9, 25 is:

(a) 2:1 (b) 5:1

(c) 7:15 (d) 3:5 **(1 mark)**

Answer:

(b) The third proportion of 12,30

$c = \frac{b^2}{a} \quad \frac{(30)^2}{12} \quad \frac{900}{12} =$ = 75

The Mean proportion of 9,25

$b = \sqrt{ac} \quad \sqrt{9 \times 25} \quad \sqrt{225} \quad = \quad = 15$

Ratio of third proportion of 12, 30

and Mean proportion of 9, 25 = 75:15

= 5:1

2015 - Dec [86] The value of $\log_5 3 \times \log_3 4 \times \log_2 5$.

(a) 0 (b) 1

(c) 2 (d) $\frac{1}{2}$ **(1 mark)**

Answer:

(c) $\log_5 3 \times \log_3 4 \times \log_2 5$

$= \frac{\log 3}{\log 5} \quad \frac{\log 4}{\log 3} \quad \frac{\log 5}{\log 2} \quad \times$

$= \frac{\log 4}{\log 2}$

$= \frac{\log 2^2}{\log 2}$

$= \frac{2 \log 2}{\log 2} \quad = 2$

2015 - Dec [87] What number must be added to each of the numbers 10, 18, 22, 38 to make the numbers is proportion?

(a) 2 (b) 4

(c) 8 (d) None of these. **(1 mark)**

Answer:

(a) Let x to be added

Then (10 + x), (18 + x), (22 + x), (38 + x) are in prop.

Product of Extremes = Product of Mean

(10 + x) (38 + x) = (18 + x) (22 + x)

$380 + 10x + 38x + x^2 = 396 + 18x + 22x + x^2$

48x + 380 = 396 + 40x

48x - 40x = 396 - 380

8x = 16

x = 2

2015 - Dec [88] The value of $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$ is:

(a) $\frac{1}{2}$ (b) $\frac{3}{2}$

(c) $\frac{2}{3}$ (d) 2

(1 mark)

Answer:

(b) $\frac{2^n + 2^{n-1}}{2^{n+1} - 2^n}$ $\frac{2^n + 2^n.2^{-1}}{2^n.2^1 - 2^n}$

$= \frac{2^n(1 + 2^{-1})}{2^n(2^1 - 1)}$

$= \frac{\left(\frac{1}{1} + \frac{1}{2}\right)}{(2 - 1)}$

$= \frac{\left(\frac{2 + 1}{2}\right)}{1}$

$= \left(\frac{3}{2}\right)$

2016 - June [89] The integral part of a logarithm is called _____ and the decimal part of a logarithm is called _____.

(a) Mantissa, Characteristic (b) Characteristic, Mantissa

(c) Whole, Decimal (d) None of these. **(1 mark)**

Answer:

(b) The integral part of a logarithms is called **Characteristic** and the decimal part of a logarithm is called **mantissa**.

2016 - June [90] The value of $\left[\frac{x^2-(y-z)^2}{(x+z)^2-y^2}+\frac{y^2-(x-z)^2}{(x+y)^2-z^2}+\frac{z^2-(x-y)^2}{(y+z)^2-x^2}\right]$

(a) 0 (b) 1

(c) – 1 (d) **(1 mark)**

Answer:

(b) $\frac{x^2-(y-z)^2}{(x+z)^2-y^2}$ $\frac{y^2-(x-z)^2}{(x+y)^2-z^2}$ + $\frac{z^2-(x-y)^2}{(y+z)^2-x^2}$ +

$=\frac{(x+y-z)(x-y+z)}{(x+z+y)(x+z-y)}$ $\frac{(y+x-z)(y-x+z)}{(x+y+z)(x+y-z)}$ + $\frac{(z+x-y)(z-x+y)}{(y+z+x)(y+z-x)}$ +

$=\frac{x+y-z}{x+y+z}$ $\frac{y+z-x}{x+y+z}$ + $\frac{z+x-y}{x+y+z}$ +

$=\frac{x+y-z+y+z-x+z+x-y}{x+y+z}$

$=\frac{x+y+z}{x+y+z}$ = 1

2016 - June [91] X, Y, Z together starts a business. If X invests 3 times as much as Y invests and Y invests two third of what Z invests, then the ratio of capitals of X, Y, Z is:

(a) 3:9:2 (b) 6:3:2

(c) 3:6:2 (d) 6:2:3 **(1 mark)**

Answer:

(d) Given x = 3y and y = $\frac{2}{3}$ z

$\frac{x}{y}$ $\frac{3}{1}$ and $\frac{y}{z}$ $\frac{2}{3}$ =

x : y = 3 : 1 and y : z = 2 : 3

= 3 × 2 : 1 × 2

= 6 : 2

x : y : z = 6 : 2 : 3

2016 - June [92] If $\log_4(x^2 + x) - \log_4(x+1) = 2$, then the value of X is:

(a) 2 (b) 3

(c) 16 (d) 8 **(1 mark)**

Answer:

(c) If $\log_4 (x^2 + x) - \log_4 (x + 1) = 2$

$$\log_4\left\{\frac{(x^2+x)}{(x+1)}\right\} = 2$$

$$\log_4\left\{\frac{x(x+1)}{(x+1)}\right\} = 2$$

$$\log_4 x = 2$$

$$x = 4^2$$

$$x = 16$$

2016 - June [93] Value of $\frac{1}{\log_3^{60}} + \frac{1}{\log_4^{60}} + \frac{1}{\log_5^{60}}$ is :

(a) 0 (b) 1

(c) 5 (d) 60 **(1 mark)**

Answer:

(b) $\frac{1}{\log_3 60} + \frac{1}{\log_4 60} + \frac{1}{\log_5 60}$

$= \log_{60} 3 + \log_{60} 4 + \log_{60} 5 \quad \left[\because \frac{1}{\log_a b} = \log_b a\right]$

$= \log_{60}(3 \times 4 \times 5)$

$= \log_{60} 60$

$= 1$

2016 - June [94] If $3^x = 5^y = 75^z$, then

(a) $x + y - z = 0$ (b) $\frac{2}{x} \frac{1}{y} \frac{1}{z} =$

(c) $\frac{1}{x} \frac{2}{y} \frac{1}{z} =$ (d) $\frac{2}{x} \frac{1}{z} \frac{1}{y} =$ **(1 mark)**

Answer:

(c) If $3^x = 5^y = 75^z = k$ (let)

then $3^x = k$, $5^y = k$, $75^z = k$

$3 = k^{1/x}$, $5 = k^{1/y}$, $75 = k^{1/z}$

we know that

$75 = 3 \times 5 \times 5$

$k^{\frac{1}{z}} \quad k^{\frac{1}{x}} \quad k^{\frac{1}{y}} \quad k^{\frac{1}{y}}$.

$k^{\frac{1}{z}} \quad k^{\frac{1}{x}+\frac{1}{y}+\frac{1}{y}}$

on comparing

$\frac{1}{z} = \frac{1}{x} + \frac{1}{y} + \frac{1}{y}$

$\frac{1}{z} = \frac{1}{x} + \frac{2}{y}$

$\frac{1}{x} + \frac{2}{y} \quad \frac{1}{z} =$

2016 - Dec [95] If log 2 = 0.3010 and log 3 = 0.4771, then the value of log 24 is:

(a) 1.0791 (b) 1.7323

(c) 1.3801 (d) 1.8301 **(1 mark)**

Answer:

(c) If log 2 = 0.3010 and log 3 = 0.4771

then log 24 = log (2 × 2 × 2 × 3)

= log 2 + log 2 + log 2 + log 3

= 3 log 2 + log 3

= 3 × 0.3010 + 0.4771

= 0.9030 + 0.4771

= 1.3801

2016 - Dec [96] If abc = 2, then the value of $\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{1}{2}b+c^{-1}} + \frac{1}{1+c+a^{-1}}$ is:

(a) 1 (b) 2

(c) 3 (d) $\frac{1}{2}$ **(1 mark)**

Answer:

(a) If abc = 2

$ab = \frac{2}{c} = 2c^{-1}$ $a = \frac{2}{bc} = 2b^{-1}c^{-1}$

$bc = \frac{2}{a} = 2a^{-1}$ $b = \frac{2}{ca} = 2c^{-1}a^{-1}$

$ca = \frac{2}{b} = 2b^{-1}$ $c = \frac{2}{ab} = 2a^{-1}b^{-1}$

Given $\frac{1}{1+a+2b^{-1}} + \frac{1}{1+\frac{1}{2}b+c^{-1}} + \frac{1}{1+c+a^{-1}}$

$= \frac{1}{1+a+2b^{-1}} + \frac{2b^{-1}}{2b^{-1}(1+\frac{1}{2}b+c^{-1})} + \frac{a}{a(1+c+a^{-1})}$

$= \frac{1}{(1+a+2b^{-1})} + \frac{2b^{-1}}{2b^{-1}+1+2b^{-1}c^{-1}} + \frac{a}{a+ac+1}$

$= \frac{1}{1+a+2b^{-1}} + \frac{2b^{-1}}{2b^{-1}+1+a} + \frac{a}{a+2b^{-1}+1}$

$= \frac{1+2b^{-1}+a}{1+a+2b^{-1}}$

$= 1$

2016 - Dec [97] There are total 23 coins of ₹ 1, ₹ 2 and ₹ 5 in a bag. If their value is ₹ 43 and the ratio of coins of ₹ 1 and ₹ 2 is 3:2. Then the number of coins of ₹ 1 is:

(a) 12 (b) 5

(c) 10 (d) 14 **(1 mark)**

Answer:

(a) Total no. of coins = 23

Ratio of ₹ 1 coin : ₹ 2 coins = 3 : 2

let No. of ₹ 1 coins = 3x

No. of ₹ 2 coins = 2x

No. of ₹ 5 coins = 23– 3x– 2x

= 23– 5x

Total value of all coins = 43

$3x \times 1 + 2x \times 2 + (23 - 5x)\ 5 = 43$

$3x + 4x + 115 - 25x = 43$

$-18x = 43 - 115$

$-18x = -72$

$x = \frac{-72}{-18} = 4$

No. of ₹ 1 coins = $3x = 3 \times 4 = 12$

2017 - June [98] If a : b = 2 : 3, b : c = 4 : 5 and c : d = 6 : 7, then a : d is:

(a) 24 : 35 (b) 8 : 15

(c) 16 : 35 (d) 7 : 15 **(1 mark)**

Answer:

(c) a : b = 2 : 3 $\frac{a}{b} \quad \frac{2}{3}$ _____(i)

b : c = 4 : 5 $\frac{b}{c} \quad \frac{4}{5}$ _____(ii)

c : d = 6 : 7 $\frac{c}{d} \quad \frac{6}{7}$ _____(iii)

Multiply equation (i) & (ii) & (iii)

$\frac{a}{b} \times \frac{b}{c} \times \frac{c}{d} \times \frac{2}{3} \times \frac{4}{5} \times \frac{6}{7} \times \frac{16}{35} \times$ =

2017 - June [99] The value of log $(1^3 + 2^3 + 3^3 + ...n^3)$ is equal to:
(a) 3 log 1 + 3 log 2 + ... + 3 log n
(b) 2 log n + 2 log (n+1) – 2 log 2
(c) log n + log (n+1) + log (2n+1) – log 6
(d) 1 **(1 mark)**

Answer:

(b) log $(1^3 + 2^3 + 3^3 + ------+ n^3)$

$= \log (\Sigma n^3)$

$= \log \left[\frac{n(n+1)}{2}\right]^2$

$= 2 \log \left[\frac{n(n+1)}{2}\right]$

$= 2 [\log n + \log (n + 1) - \log 2]$

$= 2 \log n + 2 \log (n + 1) - 2 \log 2$

2017 - June [100] If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$ then the value of $\frac{1}{a^2} + \frac{1}{b^2}$ equal to:
(a) 480 (b) 482
(c) 484 (d) 486 **(1 mark)**

Answer:

(b) If $a = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}$ and $b = \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$a + b = \frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}} + \frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}$

$= \frac{(\sqrt{6}+\sqrt{5})^2 + (\sqrt{6}-\sqrt{5})^2}{(\sqrt{6}-\sqrt{5})(\sqrt{6}+\sqrt{5})}$

$= \frac{6+5+2\sqrt{30}+6+5-2\sqrt{30}}{(\sqrt{6})^2-(\sqrt{5})^2}$

$= \frac{22}{6-5} = \frac{22}{1} = 22$

$$a \cdot b = \left(\frac{\sqrt{6}+\sqrt{5}}{\sqrt{6}-\sqrt{5}}\right)\left(\frac{\sqrt{6}-\sqrt{5}}{\sqrt{6}+\sqrt{5}}\right) = 1$$

$$\frac{1}{a^2} + \frac{1}{b^2} = \frac{b^2+a^2}{a^2 b^2} = \frac{(a+b)^2-2ab}{(ab)^2}$$

$$= \frac{(22)^2-2\times 1}{(1)^2} = \frac{484-2}{1} = 482$$